Measurement and Control of Microwave Frequencies by Lower Radio Frequencies[*]

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Summary—**From the fields of nuclear and paramagnetic resonance comes a relation between precession frequency and magnetic field strength for nuclei and unpaired electrons. The relation is such that $f_n = K_n H$ for nuclei and $f_e = K_e H$ for electrons. Thus if the frequency of one oscillator is set for f_n and the frequency of another oscillator is adjusted so that simultaneous nuclear and electronic resonance occurs in the same magnetic field, the frequency ratio of the oscillators is given by the ratio of K_e to K_n. Values of K_e and K_n have been tabulated for many substances and therefore allow frequency comparisons to be made. For example, protons in mineral oil and electrons in hydrogen have a precession frequency ratio of 658.228; hence for an f_e in x band, f_n is about 14 mc when the magnetic field is 3300 Gauss. Changing the value of H causes the frequencies to move up or down the frequency scale but their ratio is always constant. By this method microwave frequencies may be measured with equipment of a much lower frequency range. The precision of measurement is limited by the widths of the nuclear and electronic resonance curves and runs between one part in 10^4 to 10^5. This frequency measurement method may be made the basis of automatic control of microwave frequencies by quartz crystals or very stable lower frequency oscillators. An experimental model of such a system has been constructed and operated.**

INTRODUCTION

THE most well-known method of accurately measuring, generating, or controlling microwave frequencies today is that of frequency multiplication. This method usually multiplies the frequency of a stable crystal controlled oscillator in several steps, utilizing lumped constant circuits, distributed constant circuits, and finally klystron or crystal multiplier stages depending upon the final microwave power required. Many stages are required to reach the microwave region as small multiplication factors must be used in each stage if sufficient power is to be available for driving a succeeding stage. The result is a complex device composed of many circuits, each of which requires careful tuning and adjustment. The maintenance of a multiplier chain usually requires auxillary test equipment capable of covering the frequency spectrum from the crystal oscillator to final microwave output.

The Nuclear-Electronic method of measurement and control to be described here avoids many of the difficulties of the multiplier chain, but is perhaps no less complex. Its attractiveness lies in the ability to perform a frequency multiplication of some 650 times in one stage. This means that, for example, one crystal controlled oscillator circuit at 14 mc can permit measurement or control of a klystron oscillator at approximately 9300 mc. This is accomplished by comparing the precession frequencies of atomic nuclei and electrons in a magnetic field. The comparison is made visually for measurement purposes and automatically for frequency control purposes.

MAGNETIC RESONANCE

The method of frequency comparison to be described makes use of some of the properties of matter at the subatomic level. In fact, it is the properties of electrons and atomic nuclei that are of interest. The field of study concerned with the interaction of these particles and electromagnetic fields is known as magnetic resonance. When nuclei are studied, the term nuclear magnetic resonance or nuclear magnetism is used; when electron interactions are studied the term paramagnetic or ferromagnetic resonance may be used. Since electrons and nuclei are predominantly found combined to form matter, material substances may be expected to, and indeed do perform magnetic resonance.

Resonance implies a tuning or a matching between an applied frequency and a frequency either actually or potentially present in the substance in question. A violin string, a cymbal, the air column in an organ pipe, an electrical circuit comprising capacity and inductance, all resonate to the frequency which is that of their own natural vibrations. There is a frequency peculiar to electrons and some nuclei which is matched by an applied frequency when magnetic resonance occurs. This natural frequency is not, however, one of vibration, but is a frequency of precession. The mechanism of precession may be recalled by considering the spinning top or gyroscope of Fig. 1. The top has angular momentum along its axis of rotation. Gravity will apply a force to the spinning top through the center of gravity, and the floor the equal and opposite reaction. The result of this torque is a precession about lines of gravitational force.

A similar situation exists for electrons and some nuclei when placed in a magnetic field. Electrons and some nuclei have rotation about an axis through their centers and therefore have angular momentum. Because nuclei are composed of several spinning particles (protons and neutrons) it is possible that the resultant angular momentum may be zero. In fact, only about half of the known nuclei have a rotation of the nucleus as a whole about an axis which passes through its center of gravity. Since electrons and protons are charged particles they are in effect a small current by virtue of their motion. This current may be considered to cause a magnetic field to exist along the axis of rotation. The nucleus or electron is thus said to possess a magnetic moment

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[*] Manuscript received by the PGMTT, June 15, 1956. Supported in part by contract N6-onr-27519.

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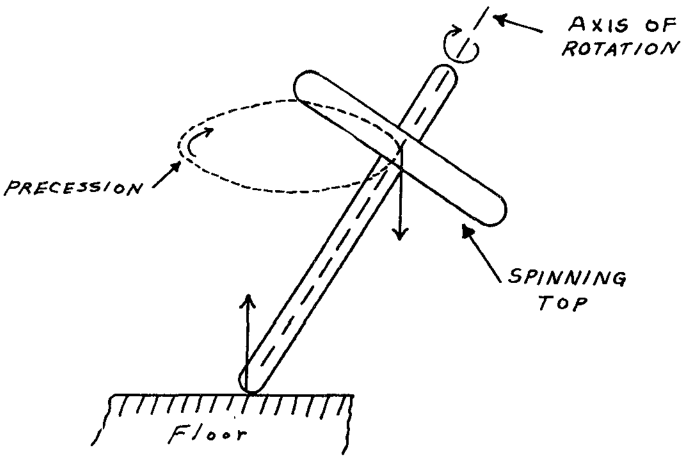

Fig. 1—Top spinning in a gravitational field.

much the same as a small bar magnet. Consider a single nucleus located between the poles of a magnet. The essential features of the situation are illustrated in Fig. 2. Because of its magnetic moment, the spinning nucleus behaves as if a small bar magnet were rigidly attached to it and oriented in the direction of the axis of rotation. The north pole of this equivalent bar magnet will be repelled from the north pole of the magnet by a downward force, and the south pole will be repelled from the south pole of the magnet by an upward force. These two equal and opposite forces result in a torque upon the nucleus. The result is a precession of the axis of the spinning nucleus and the magnetic moment. The rate at which this precession occurs is proportional to the strength of the magnetic field existing at the nucleus. The factor of proportionality is characteristic of the material containing the nuclei as the field at a nucleus is the resultant of the applied external field and the internal field due to neighboring atoms. The same situation exists for an electron placed in a magnetic field except that for the same field strength the precession frequency is greater, resulting in a larger constant of proportionality.

Electrons are the commonest of particles found in the universe, but they display magnetic resonance only in certain special substances. The spin and magnetic moment properties are elusive and not commonly observed because most matter is made up of electrons that pair off two by two in such a way that, as far as external measurements are concerned, their effects are cancelled out. The exceptional cases are those of certain free atoms, ferromagnetic substances, and some paramagnetic substances which usually contain an odd number of electrons. The odd number means that at least one electron must be unpaired and its undiminished magnetic properties may be detected by external measurements.

When a material containing nuclei having a magnetic moment or unpaired electrons is placed in a magnetic field, all of these particles precess about the field lines at the gyromagnetic frequency but in random phase so that the net effect external to the sample is zero. If the precessing particles are, in addition, placed in an alternating magnetic field of the same frequency, they may be made to fall in step with the alternating field. This is the magnetic resonance condition—external applied field of the same frequency as the precessing magnetic moment For maximum effect, the oscillating field must lie in a plane perpendicular to the lines of magnetic force about which the magnetic moments are precessing. Changing the random phase precession to a coherent one takes some energy from the applied alternating magnetic field. This loss of energy in the applied alternating field is a means by which magnetic resonance of the particles is detected.

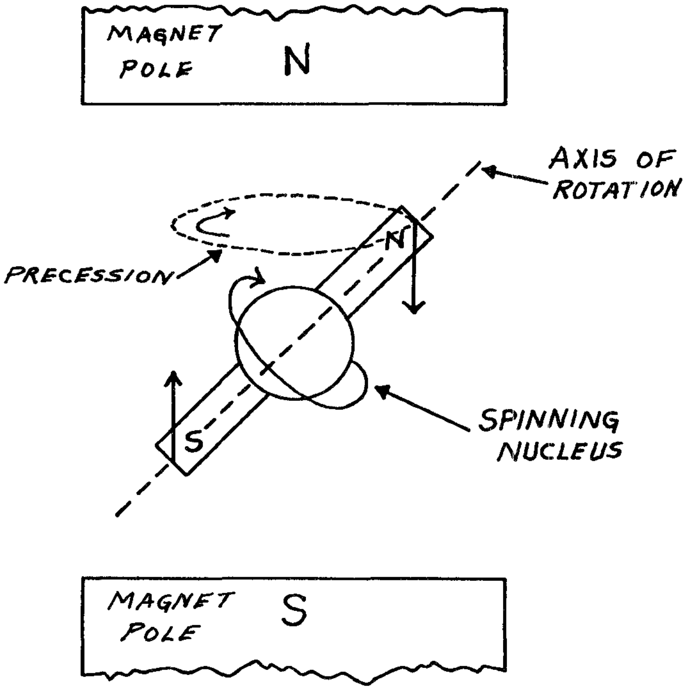

Fig. 2—Nucleus spinning in a magnetic field.

Nuclear Resonance

The components necessary in a system for detecting nuclear resonance absorption will now be described in general (see Fig. 3). The basic requirement is that of a strong steady magnetic field and a comparatively weak oscillating magnetic field of the proper frequency oriented perpendicular to one another. The steady field may be supplied by either a permanent magnet or an electromagnet. In the experimental setup a surplus 10 cm magnetron magnet was used at a field strength of about 3300 Gauss. The sample consisted of hydrogen nuclei or protons in water which resonate in the above field at about 14 mc. The coil which supplied the alternating rf field to the sample is the tank coil of an oscillator operating at the precession frequency. The amplitude of the oscillations decreases when resonance occurs due to the energy absorption of the sample. To facilitate observation of the resonance condition the steady mag-

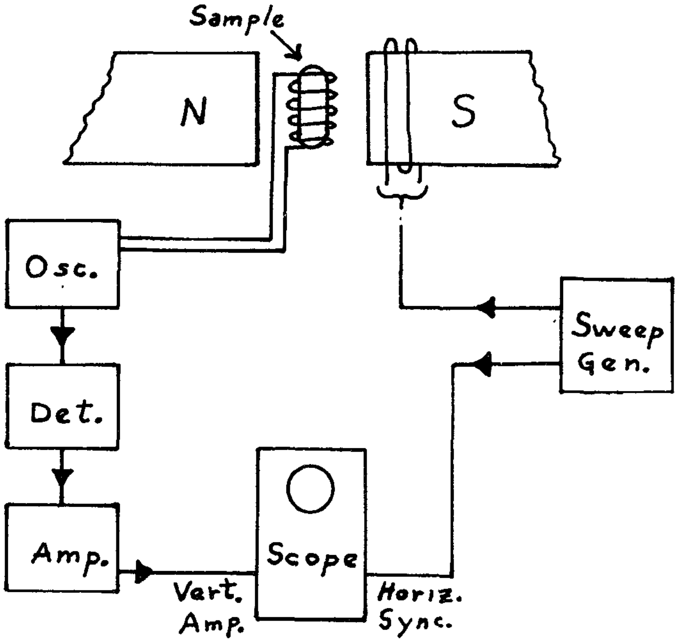

Fig. 3—System for detecting nuclear resonance.

netic field is amplitude modulated by an audio frequency about 15 Gauss from its steady resonance value. This causes the magnetic resonance to be swept through twice each audio cycle. The resonance may be observed visually if the oscillator amplitude is displayed on the vertical axis of an oscilloscope when the horizontal sweep is locked in synchronism with the magnetic field sweep frequency. The resonance condition is easier to observe if the horizontal sweep frequency is just half of the magnet sweep frequency. Fig. 4 shows how the scope pattern arises for H_{DC} exactly at resonance and for H_{DC} slightly above resonance. At resonance the four peaks will be equally spaced.

Electronic Resonance

The magnetic resonance of electrons requires the same perpendicular relation between the steady magnetic field and the oscillating rf field. For the same magnetic field as in the nuclear resonance case (3300 Gauss) the required frequency for electronic resonance is in the X-band microwave region at 9300 mc. This requires that the sample containing unpaired electrons be placed in a microwave cavity resonant to 9300 mc. The general picture is shown in Fig. 5. The sample must be placed in the cavity at a position of maximum rf magnetic field, and the cavity must be oriented so the oscillating microwave magnetic field is perpendicular to the steady magnetic field. If the cavity is of the transmission type, detection of the energy passing through it will give the resonance information. Again this signal is applied to the vertical axis of an oscilloscope with horizontal sweep synchronized to the magnetic sweep frequency. The resonance display is the same as in the nuclear case.

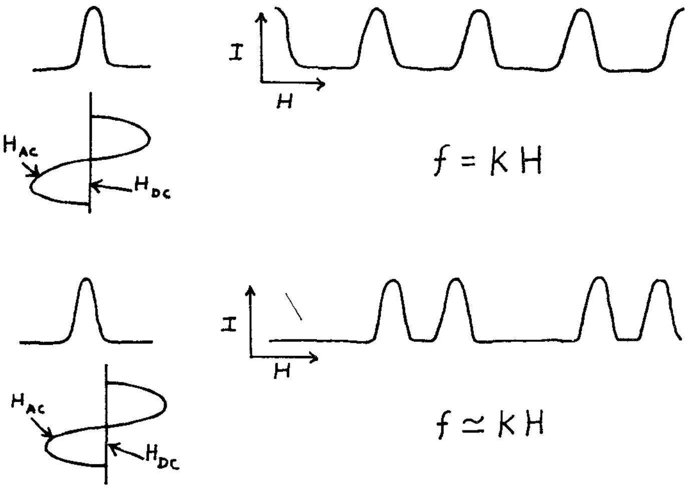

Fig. 4—Oscilloscope pattern of magnetic resonance.

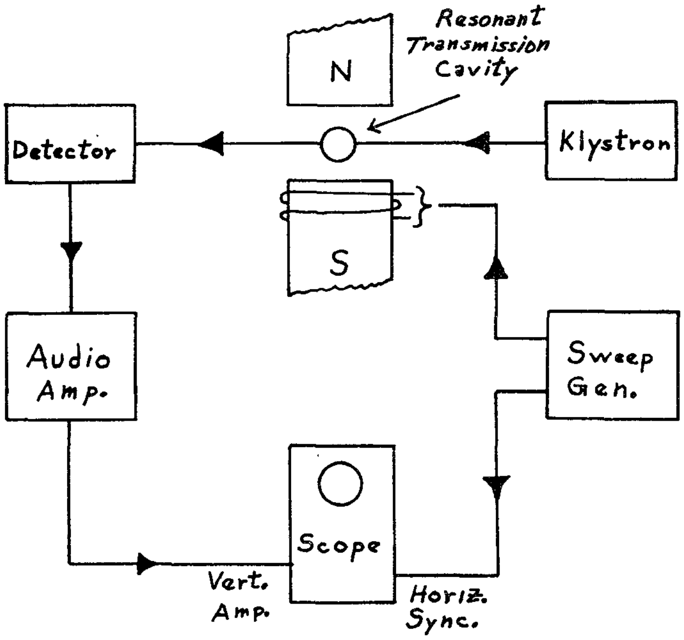

Fig. 5—System for detecting electron resonance.

Field-Frequency Relation

As was previously stated, the precession frequency is proportional to the field existing at the particle in question. This is simply expressed here for our purposes as

$$f = kH. \tag{1}$$

The constant of proportionality is characteristic of the particle, nucleus or electron, and its environment, *i.e.*, fields due to surrounding particles. For protons in water $k \simeq 4.3$ kc/Gauss and for almost free electrons $k \simeq 2.8$ mc/Gauss. Values of k have been tabulated for many nuclei and for electrons in many substances. The fact that k is accurately known for many nuclei has led to the use of nuclear resonance for magnetic field measurement.

Note that (1) allows considerable freedom in the value of frequency or field at resonance. If for some reason the field available is limited, the frequency may be adjusted according to (1) to cause resonance within the range of available field strength. However, the resonance signal increases with increasing field strength and this factor may place a lower limit on H_{DC}. Values of H_{DC} range in practice from a few to ten thousand Gauss. Fields of good homogeneity stronger than 10^4 Gauss are difficult to achieve.

SIMULTANEOUS RESONANCE

Consider now a nuclear and electron resonance taking place in the same magnet. The following relations exist

$$f_n = k_n H \qquad \text{nuclear resonance} \tag{2}$$

$$f_e = k_e H \qquad \text{electron resonance} \tag{3}$$

$$f_e = \frac{k_e}{k_n} f_n \qquad \text{simultaneous resonance.} \tag{4}$$

Eq. (4) states that the ratio of the two oscillator frequencies is the same as the ratio of the two proportionality constants. Thus if simultaneous resonance is observed and one frequency is measured, the other frequency can be computed. Using the aforementioned values of k_e and k_n, (4) becomes

$$f_e = \frac{2.8\,\text{mc/g}}{4.3\,\text{kc/g}} f_n, \quad \text{or,} \quad f_e \cong 650 f_n. \tag{5}$$

Thus in this one comparison stage frequencies differing by a factor of some 650 times may be measured and compared. Again using the aforementioned values for k_e, k_n, and H, a radio frequency of 14 mc simultaneously resonates with a microwave frequency of 9300 mc. If the magnet field strength is doubled—raised to 6600 Gauss —then 28 mc will resonate with 18.6 kmc. Thus by changing the magnetic field strength a considerable range of frequencies may be covered. Further range extension may be obtained by choosing a ratio of k_e/k_n to be greater or smaller as needed.

The implication here is that frequencies in the microwave range may be measured or controlled by lower radio frequencies by simultaneous nuclear and electronic resonance in the same magnetic field. The accurate measurement of radio frequencies in the 1–50 mc range no longer presents a problem. A block diagram of a system for observing simultaneous resonance is shown in Fig. 6. Fig. 7 shows an oscillograph display of simultaneous resonance of protons in water (upper trace) and electrons in diphenyl-trinitrophenyl-hydrazyl (lower trace). The exact frequency ratio between the two traces is 658.66.

The reverse procedure is used to determine the proportionality constant of a material; the two frequencies are measured and knowing the constant for one material, the unknown constant is computed.

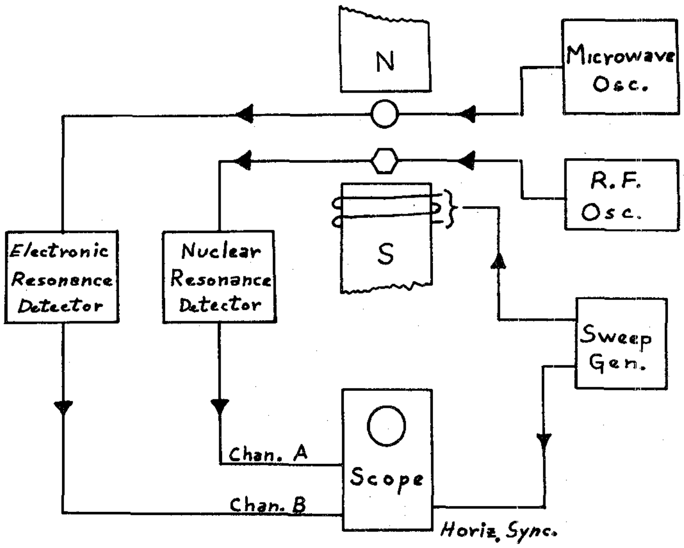

Fig. 6—Setup for observing simultaneous nuclear and electronic resonance.

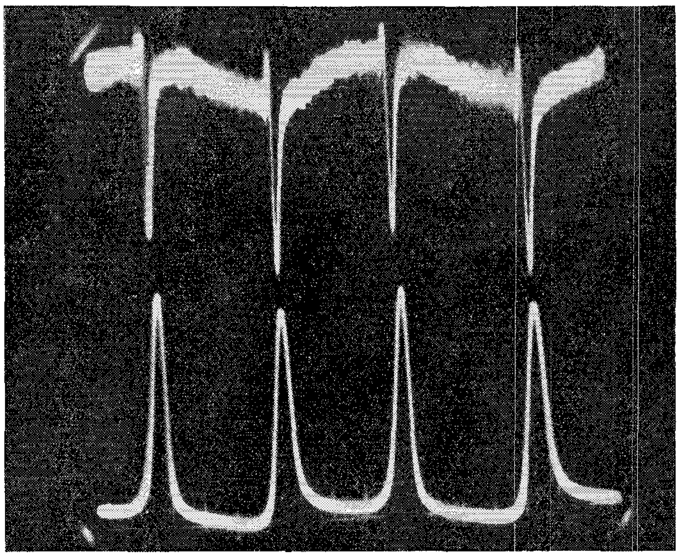

Fig. 7—Oscillographic display of simultaneous resonance. Upper trace—nuclear resonance of protons in water. Lower trace—electronic resonance of hydrazyl.

AUTOMATIC FREQUENCY CONTROL

Automatic frequency control of a microwave signal is possible with the simultaneous comparison stage. The general method is as follows (see Fig. 8): a crystal controlled or otherwise very stable oscillator operating at 14 mc causes nuclear resonance of protons in a magnetic field at about 3300 Gauss. A klystron oscillator causes an electron resonance at 9300 mc in the same magnet. The two resonance signals are fed into an electronic circuit which checks simultaneity and produces an output voltage of one sign for microwave frequency too high and a voltage of opposite sign for microwave frequency too low. This error signal is fed to a klystron control electrode, preferably the reflector, so as to cor-

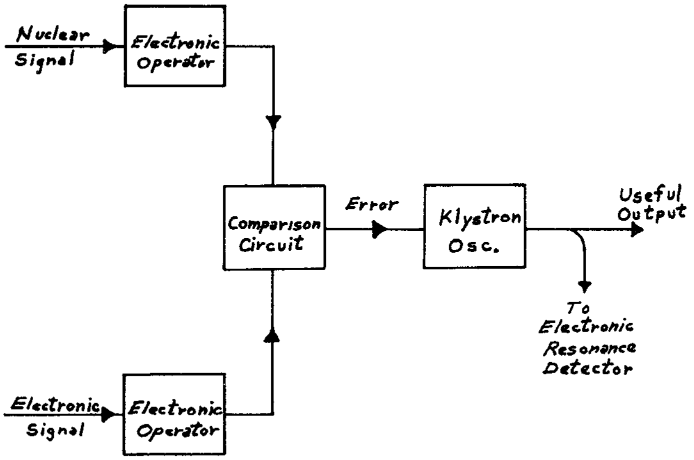

Fig. 8—AFC system using simultaneous resonance.

rect the microwave frequency for simultaneity. A strong electronic resonance signal may be observed with only 100 microwatts of microwave power leaving the major portion of the klystron power for useful output. The blocks marked electronic operator prepare the resonance signals so they may be fed into a phase detector and thus give rise to an error signal.

LIMITATIONS

The major limitation on the accuracy of measurement or control is the natural resonance line width of the electron resonance. One of the substances giving the strongest electron resonance is diphenyl-trinitrophenyl-hydrazyl with a line width of 2.7 Gauss. On the assumption that the center one tenth of this curve could be located, an accuracy of 1 part in 10^4 could be obtained. Another factor contributing to inaccuracy is magnetic field homogeneity. All of the sample must see the same field strength else the resonance line will be broadened beyond its natural width. Good field homogeneity may be insured by careful alignment of the pole faces and by using a large ratio of pole diameter to gap distance. Careful shimming of the pole face will also improve field homogeneity. Fields more homogeneous than one part in 10^6 over a square centimeter are difficult to attain because of local inhomogeneities in the magnetic properties of the pole face material.

Narrow electron resonance lines may be obtained from the electrons in an electron beam which is made to interact with the microwave field. Line widths of the order of 0.5 Gauss have been reported by this method giving a frequency ratio accurate to one part in 10^5. The narrowest electron resonance to come to our attention is that of a solution of sodium in ammonia with a half maximum width of 0.08 Gauss. This electron resonance used with a water proton resonance might give rise to control accuracies of one part in 10^6, depending on the signal to noise ratio for the electron resonance and the field homogeneity.

The observed width of the proton resonance in water is usually due to the magnetic field inhomogeneity as it has been possible to obtain in fields of 7000 Gauss proton resonance curves with a line width of 0.001 Gauss. This corresponds to a resolution of one part in 7 million.

Discontinuities in a Rectangular Waveguide Partially Filled with Dielectric*

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Summary—**The modal spectrum for a rectangular waveguide with a dielectric slab at the bottom of the guide is obtained following the Characteristic Green's Function method developed by Marcuvitz. Then a four-terminal network is found as equivalent to the junction of the partially filled waveguide and an empty rectangular waveguide.**

An integral equation is written for the electric field at the plane of the junction and variational expressions are derived for the parameters of the four-terminal network connecting the transmission line equivalent to the partially filled waveguide to the transmission line equivalent to the empty guide.

A reasonable guess for the electric field at the discontinuity gives approximate values for the parameters of the four-terminal network. These values agree with experiment.

The parameters of the network are plotted vs frequency and thickness of the slab.

INTRODUCTION

A CONSIDERABLE AMOUNT of literature has been devoted recently to surface waves. The reader is referred to van Bladel and Higgins[1] for an introduction to the effect of dielectrics in rectangular waveguides and to Barlow and Cullen[2] for surface waves in dielectric slabs. The main purpose of this paper is to obtain a four-terminal network equivalent to the junction of an empty rectangular waveguide and a rectangular waveguide partially filled with dielectric. The Schwinger variational principle combined with the

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* Manuscript received by PGMTT, June 20, 1956.

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[1] J. van Bladel and T. J. Higgins, "Cut-off frequency in two-dielectric layered rectangular wave guides," *J. App. Phys.*, vol. 22, p. 329; March, 1951.

[2] H. M. Barlow and A. L. Cullen, "Surface waves," *Proc. IEE*, Part III, vol. 100, p. 329; November, 1953.